College Entrance Examination Board Advanced Placement Examination MATHEMATICS: CALCULUS AB

SECTION II

Time-1 hour and 30 minutes

SHOW ALL YOUR WORK. INDICATE CLEARLY THE METHODS YOU USE BECAUSE YOU WILL BE GRADED ON THE CORRECTNESS OF YOUR METHODS AS WELL AS ON THE ACCURACY OF YOUR FINAL ANSWERS.

- 1. Given $f(x) = |\sin x|$, $-\pi \le x \le \pi$, and $g(x) = x^2$ for all real x.
 - (a) On the axes provided sketch the graph of f.
 - (b) Let H(x) = g(f(x)). Write an expression for H(x).
 - (c) Find the domain and range of H.
 - (d) Find an equation of the line tangent to the graph of H at the point where $x = \frac{\pi}{4}$.
- 2. Let $P(x) = x^4 + ax^3 + bx^2 + cx + d$. The graph of y = P(x) is symmetric with respect to the Y-axis, has a relative maximum at (0, 1), and has an absolute minimum at (q, -3).
 - (a) Determine the values of a, b, c, and d, and using these values write an expression for P(x).
 - (b) Find all possible values of q.
- 3. Let $f(x) = kx^2 + c$.
 - (a) Find x_0 in terms of k such that the tangent lines to the graph of f at $(x_0, f(x_0))$ and $(-x_0, f(-x_0))$ are perpendicular.
 - (b) Find the slopes of the tangent lines mentioned in (a).
 - (c) Find the coordinates, in terms of k and c, of the point of intersection of the tangent lines mentioned in (a).
- 4. Let f be a function defined for all x > -5 and having the following properties.
 - (i) $f''(x) = \frac{1}{3\sqrt{x+5}}$ for all x in the domain of f.
 - (ii) The line tangent to the graph of f at (4, 2) has an angle of inclination of 45°.

Find an expression for f(x).

- 5. A ball is thrown from the origin of a coordinate system. The equation of its path is y = mxm is positive and represents the slope of the path of the ball at the origin.
- a) For what value of m will the ball strike the horizontal axis at the greatest distance from the origin? Justify your answer.
 - (b) For what value of m will the ball strike at the greatest height on a vertical wall located 100 feet from the
- 6. Given two functions f and g defined by $f(x) = \tan x$ and $g(x) = \sqrt{2} \cos x$.
 - (a) Find the coordinates of the point of intersection of the graphs of f and g in the interval $0 < x < \frac{\pi}{2}$.
 - (b) Find the area of the region enclosed by the Y-axis and the graphs of f and g.
- ₹ 7. The rate of change in the number of bacteria in a culture is proportional to the number present. In a certain laboratory experiment, a culture had 10,000 bacteria initially, 20,000 bacteria at time t₁ minutes, and 100,000 bacteria at $(t_1 + 10)$ minutes.
 - (a) In terms of t only, find the number of bacteria in the culture at any time t minutes, $t \ge 0$.
 - (b) How many hacteria were there after 20 minutes?
 - (c) How many minutes had elapsed when the 20,000 bacteria were observed?

END OF EXAMINATION

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